

## Top anomalous magnetic moment and the two photon decay of Higgs

Lance Labun and Johann Rafelski

*Department of Physics, University of Arizona, Tucson, Arizona, 85721 USA, and  
TH Division, Physics Department, CERN, CH-1211 Geneva 23, Switzerland***Abstract**

We consider the effect of the magnetic moment of the top quark on the Higgs  $h \rightarrow \gamma\gamma$  two photon decays and we evaluate this decay amplitude allowing for an arbitrary top gyromagnetic factor  $g_t$  value. We show that for any  $g_t \neq 2$  the  $h \rightarrow \gamma\gamma$  decay rate is always enhanced: the negative interference between W-loop and top loop is reduced and even reversed. For  $g_t \rightarrow 0, \pm 4, \dots$  the enhancement of the  $h \rightarrow \gamma\gamma$  decay rate is up to a factor two. Standard model interactions' 1st order perturbative effect on  $g_t$  amounts to a 2% enhancement of the  $h \rightarrow \gamma\gamma$ . Nonperturbative and BSM contributions to  $g_t$ , and the lack of other sensitive experimental constraints are noted;  $h \rightarrow \gamma\gamma$  seems to be the most sensitive presently available probe of the top gyromagnetic factor.

*Keywords:**PACS:* 14.65.Ha, 14.80.Bn, 13.40.Em, 13.40.Hq

**Introduction:** We explore the consequence for the Higgs to two photon decay  $h \rightarrow \gamma\gamma$  of the top magnetic moment

$$\mu_t = \frac{g_t}{2} \frac{Qe}{2m_t}, \quad Qe = +(2/3)e \quad (1)$$

having a value considerably different from that obtained using the Dirac gyromagnetic ratio  $g_t \rightarrow g_D = 2$ . We consider  $g_t$  to be a parameter, which may be determined from independent experiment and theoretical studies.

Anomalous couplings of the top quark to gluons and photons have been the subject of theoretical interest [1–3] and may soon be accessible to experiments [4]. Standard model (SM) corrections to the top form factor are significant and we note the top-Higgs coupling  $e_{th}^2/4\pi \simeq 1/4\pi = 0.08$  where the minimal coupling is  $e_{th} = m_t\sqrt{2}/v \simeq 0.99$ , with Higgs vacuum expectation value  $v = 246.2 \text{ GeV} \simeq (G_F\sqrt{2})^{-1/2}$ ,  $G_F$  is the Fermi constant and  $m_t = 173.4 \text{ GeV}$ . Therefore the virtual Higgs correction competes with the strong interaction gluon-top coupling  $\alpha_s(m_t) = 0.108$  indicating that the Higgs may play a special role in understanding of the top electromagnetic structure.

It is also not surprising that the two-loop QCD contribution to the electromagnetic form factor and hence  $g_t$  is calculated to be nearly as large as the one-loop contribution [1, 2]. Interest in understanding the consequences of anomalous moments of the top both, for QED and QCD coupling, has motivated theoretical efforts since the measurement of the top's production cross section [5–7]. Much interest in the magnetic properties of the top is due to its expected sensitivity to beyond the standard model (BSM) physics [3]. The sensitivity of the value of magnetic moment to compositeness is well known [8].

The  $h \rightarrow \gamma\gamma$  decay has been studied for  $g_t = g_D$ . It

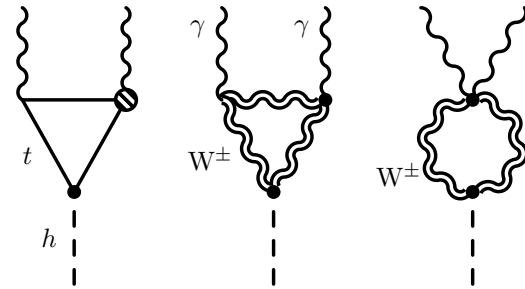


Figure 1: Diagrams giving dominant contribution to the Higgs  $h$  decay to two photons  $\gamma$ . In the top loop at left, we denote the top-photon vertex with a shaded circle to signify that we consider a general value of the top gyromagnetic factor.

arises predominantly from the fluctuations of W-bosons and the top quark [9, 10], illustrated by the triangle diagrams in Fig. 1. The contribution due to an anomalous gyromagnetic factor  $g_t$  is indicated by the large vertex. We checked that for  $g_t \rightarrow 2$  we arrive at these well known results and will show enhancement for  $g_t \neq 2$  of the total decay amplitude and decay rate.

To account for significant modifications of the Dirac value  $g_t \neq g_D$ , one must in principle change from 1st-order to 2nd-order Fermion theory resulting in additional two-top to two-photon vertices and an additional diagram not shown in figure 1, for details of the 2nd order framework perturbative computation see Ref. [11]. We can sidestep an in-depth discussion of the 2nd-order Fermion theory by following Ref. [9], that is, by using for the Higgs decay the  $\beta$ -function which we obtained using the external field method for all  $g_t$  [12]. Our  $\beta(g_t)$  function agrees in the interval  $-2 < g_t < 2$  with the perturbative result [11].

**Higgs to two photon amplitude:** The top quark loop contribution to the  $h \rightarrow \gamma\gamma$  decay has been obtained in the low energy limit,  $m_h \ll m_t$ , of the amplitude, see Eqs. (2) & (9) of [9], see also [13],

$$A_t(h \rightarrow \gamma\gamma) \simeq \frac{1}{v} \frac{\alpha b_0}{4\pi} (k_1^\kappa \epsilon_1^\lambda - k_1^\lambda \epsilon_1^\kappa)(k_2^\kappa \epsilon_2^\lambda - k_2^\lambda \epsilon_2^\kappa) \quad (2)$$

employing the  $\beta$ -function, where  $b_0$  is discussed in next paragraph.  $\alpha = e^2/4\pi$  is the electromagnetic coupling, and  $k_{1,2}^\kappa$  and  $\epsilon_{1,2}^\lambda$  are the 4-momenta and polarization vector of the photons. Evaluating the amplitude from Eq. (2) means an error of a few percent relative to the result from the exact amplitude [9, 10] for the value  $(m_h/m_t)^2 \simeq 0.52$ . Thus although  $(m_h/m_t)^2$  is not very small, this limit suffices for our first study of the dependence of the decay width on the top gyromagnetic factor  $g_t$ .

The reason for the simplification seen in Eq. (2) is that in the limit  $m_h \ll m_t$ , the contribution of the top quark loop to the  $h \rightarrow \gamma\gamma$  decay amplitude is related to top quark contribution to the renormalization group  $\beta$ -function of the electromagnetic coupling [9, 10]. The perturbative expansion of the  $\beta$ -function is

$$\beta \equiv \lambda \frac{\partial \alpha}{\partial \lambda}, \quad \beta(\alpha) = -\frac{b_0}{2\pi} \alpha^2 + \dots \quad (3)$$

The one-loop contribution to the QED  $\beta$ -function has been obtained for a fermion with arbitrary magnetic moment using two independent methods: a) perturbative computation [11] and background field method [12], and the latter method has been extended to large values of  $|g_t| > 2$  where  $b_0(g_t)$  is periodic outside the domain  $-2 < g_t < 2$ .

The above QED result for  $b_0(g_t)$  is adapted for the top by including in Eq. (9) of [12] overall factors  $Q^2$  arising from the charge of the top and  $N_c = 3$  for the color trace. Then the  $b_0$  coefficient as a function of  $g_t$  is

$$b_0(g_t) = -\frac{4}{3} N_c Q^2 \left( \frac{3}{8} g_t^2 - \frac{1}{2} \right) \quad (4)$$

The  $-4/3$  factor separated in front is the well-known value of  $b_0$  for a fermion with unit charge and  $g_t = 2$ . Eq. (4), normalized by  $b_0(2) = -(4/3)^2$ , is shown in Fig. 2.

An important observation about  $b_0(g_t)$  is that it changes sign at  $g_t = \pm 2/\sqrt{3}$  and hence is positive for  $|g_t| < 2/\sqrt{3}$ . The separate consideration of diamagnetic and paramagnetic contributions in Eq. (4) reveals that this effect is due to the decrease in strength of the paramagnetic term as  $g_t$  diminishes [14]. We thus find that in a wide range of  $g_t$ , the overall sign of the top contribution to the amplitude Eq. (2) is reversed from  $g_t = 2$  expectation, and the pattern of interference with the W-boson loop in the decay changes. For  $g_t = 2$  there is partial cancellation between the two contributions, while for  $|g_t| < \pm 2/\sqrt{3}$  the W and top loop-amplitudes add. The excess of events [15, 16] in the two photon decay channel of the Higgs was noted to correspond to a change in relative negative sign between the W and top contributions [17]. The  $g_t$ -dependent  $\beta$ -function then supplies a mechanism for this observation.

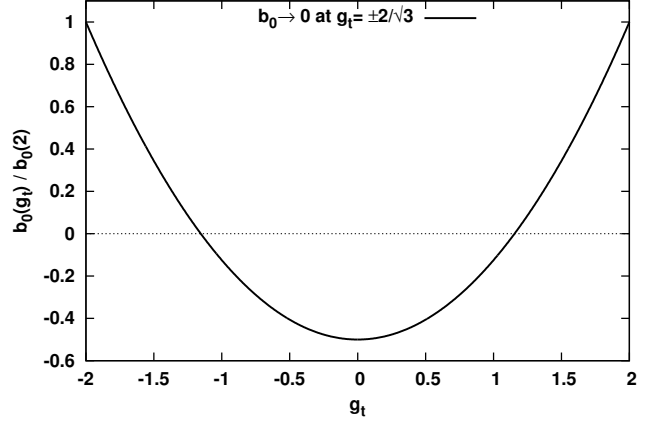


Figure 2: The  $b_0$  coefficient of the electromagnetic  $\beta$  function contributed by the top quark normalized to its value at Dirac  $g_D = 2$ .

**$g_t$ -dependent decay amplitude:** The dependence of the total  $h \rightarrow \gamma\gamma$  decay amplitude on the top magnetic moment is found by combining the  $g$ -dependent top-loop contribution with the contribution from the W-boson loop. Inserting the  $g$ -dependent  $\beta$  function Eq. (4) in Eq. (2), the total amplitude for Higgs decay into two photons is

$$A_{\text{tot}}(h \rightarrow \gamma\gamma) \simeq A_t(h \rightarrow \gamma\gamma) + A_W(h \rightarrow \gamma\gamma) \quad (5)$$

with the W loop contributing [9, 10]

$$A_W(h \rightarrow \gamma\gamma) = f_W \frac{1}{v} \frac{\alpha}{4\pi} (k_1^\kappa \epsilon_1^\lambda - k_1^\lambda \epsilon_1^\kappa)(k_2^\kappa \epsilon_2^\lambda - k_2^\lambda \epsilon_2^\kappa) \quad (6)$$

$$f_W(x) = 3x(2-x) \left( \arcsin(x^{-1/2}) \right)^2 + 3x + 2, \quad x = \frac{4m_W^2}{m_h^2}$$

for which we have stated only the relevant case  $4m_W^2/m_h^2 \equiv x > 1$  from [9], and we use  $x = 1.641$  corresponding to  $m_h = 125.5$  GeV.

Fig. 3 shows the  $g_t$  dependence of the total  $h \rightarrow \gamma\gamma$  decay amplitude, normalized to its value at  $g_t = g_D = 2$ . Considering the periodicity of  $b_0(g_t)$  the decay amplitude  $h \rightarrow \gamma\gamma$  is always enhanced as compared to  $g = g_D$  (and its periodic recurrences i.e.  $g_t = \pm 2, \pm 6, \pm 10$  etc).

To compare with the measured decay rate, we evaluate the decay width (Eq. (3) of [9] and Eq. (7) of [10])

$$\Gamma_{h \rightarrow \gamma\gamma} \simeq |f|^2 \left( \frac{\alpha}{4\pi} \right)^2 \frac{m_h^3}{16\pi v^2}, \quad f = f_W \left( \frac{4m_W^2}{m_h^2} \right) + b_0(g_t) \quad (7)$$

with  $f_W(x)$  given in Eq. (6) and  $b_0(g)$  from Eq. (4). Equation (7) provides for  $g_t \rightarrow 2$  the Higgs to two photon decay rate within a few percent of the width stated in the 2011 updated Higgs Cross Section Working Group tables (which include NLO QCD and electroweak corrections), after accounting for partial decay widths  $h \rightarrow ZZ$ ,  $h \rightarrow WW$  and to 4-fermions, see Eq. (1) of [18].

At present, there is tension in the two photon decay rate of the Higgs candidate, (see ATLAS results, Table 9

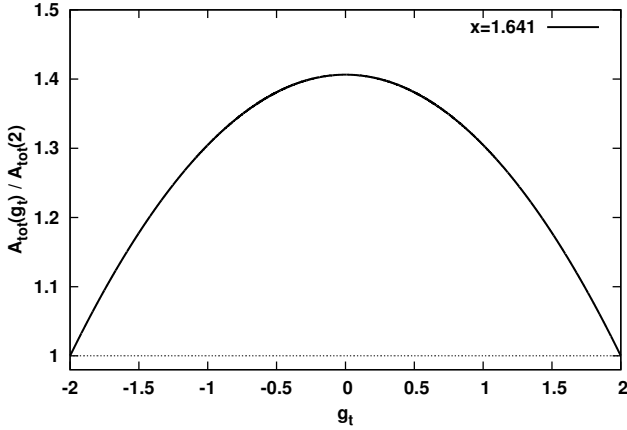


Figure 3: Higgs to two photons  $h \rightarrow \gamma\gamma$  decay amplitude normalized to its value at  $g_t = 2$ , i.e.  $A_{h \rightarrow \gamma\gamma}(g_t)/A_{h \rightarrow \gamma\gamma}(2)$  for  $x = 4m_W^2/m_h^2 = 1.64$ ,  $m_h = 125.5$  GeV.

of [15] and CMS [16]). As long as the decay rate enhancement turns out not to be larger than 2, it can be explained solely by a variation of the gyromagnetic factor from the Dirac value  $g_t = g_D$ . Currently, there is indication of rate enhancement by up to a factor 2, which would imply that the top could have a very small gyromagnetic factor  $g_t$ .

**What we know about value of  $g_t$ :** The lowest order perturbative calculation of  $g_t$  yields at one loop three contributions: a) the standard QED result, b) a similar QCD result [1, 2], and c) electroweak contributions:

$$(g_t - 2)_{\text{QED}}^{(1L)} = \alpha_{\text{QED}}/\pi = 2.5 \cdot 10^{-3} \quad (8a)$$

$$(g_t - 2)_{\text{QCD}}^{(1L)} = \frac{N_c^2 - 1}{2N_c} \frac{\alpha_s}{\pi} = 25 \cdot 10^{-3} \quad (8b)$$

$$(g_t - 2)_{\text{EW}}^{(1L)} = 7.5 \cdot 10^{-3}. \quad (8c)$$

To obtain the numerical values of contributions to  $g_t - 2$ , all couplings have to be taken at the top-scale. Calculating the amplitude Eq. (5), one expects from these usual perturbative corrections not more than a 1.0% top decay amplitude modification and hence a 2.0% enhancement of the decay rate Eq. (7). To check  $(g_t - 2)_{\text{EW}}^{(1L)}$  again we note that all vertex corrections have the same loop integral. Therefore, we can use Eq. (3) of [19] in combination with their Eq. (1) which identifies their  $\Delta\kappa \rightarrow g_t - 2$ .

There is a further important contribution of higher order QCD and virtual Higgs which have not been considered. However, it is possible to conclude that the perturbative value of  $g_t - 2$  predicts a much too small modification (a few percent) of decay rate to explain the observed tension in the  $h \rightarrow \gamma\gamma$  decay rate. Similarly, order of magnitude consideration of compositeness yield  $(g_t - 2)/2 \sim m_t/M_{\text{comp}}$  [8] and suggest a compositeness scale  $M_{\text{comp}}$  near to the top scale  $m_t$  to produce a large enhancement. These remarks do not take into account enhancements originating in other BSM physics or in non-perturbative

Higgs-top structure, arising in view of the strong Higgs-top minimal coupling.

Experimental input on the magnitude of  $g_t$  comes from the radiative  $b \rightarrow s\gamma$  decay [20, 21], recently updated, see Eq. (9) in [4]

$$-1.83 < \Delta\mu_t m_t < 0.53, \quad \frac{\Delta\mu_t}{2} = \frac{g_t - 2}{2} \frac{\frac{2}{3}e}{2m_t} \quad (9)$$

In above we have neglected the electric dipole moment term, which is constrained to be  $\sim 10^6$  times smaller than  $\mu_t$ . We consider the numbers in Eq. (9) of [4] dimensionless and translating into a bound on  $g_t$ ,

$$-3.49 < g_t < 3.59 \quad (10)$$

This constraint is thus not relevant in our consideration, as it is consistent with the full range of  $g_t$  factors suggested by the enhancement of the rate shown in figure. 3.

Limits on  $g_t$  should arise from the study of top production. In hadron colliders, top production is predominantly via strong interactions, so the LHC is more sensitive to the top chromomagnetic moment, i.e. the corresponding  $g_t^c$  ( $c$  for chromodynamic factor) [22], and  $g_t^c$  could perhaps be connected using Schwinger-Dyson equations to  $g_t$ . However, only a future direct study of the top production in  $e^+e^-$  collisions appears to offer another sensitive measure of the QED top anomalous magnetic dipole moment [23].

**Discussion and conclusions:** We have taken the top  $g_t$  factor as a parameter, which arguably could be quite different from the tree level Dirac  $g_t = g_D = 2$  value, and have shown that  $g_t$  can have a significant impact on the Higgs decay rate into two photons. The new result we have presented is the explicit dependence of the Higgs decay width to two photons on the top gyromagnetic factor  $g_t$ , Eq. (7), with Eq. (4) generalizing the required top-loop amplitude.

We have shown that for  $g_t \rightarrow 0$  the rate of Higgs to two photon decay  $h \rightarrow \gamma\gamma$  can increase by up to a factor two. We find that in the interval  $-2 < g_t < 2$  the deviation from the Dirac value  $g_t = 2$  always enhances the Higgs decay rate. In fact, any anomalous magnetic moment (that is  $g_t \neq 2$  with period 4) of the top always enhances the rate, due to the enhancement of the decay amplitude for all values  $-2 < g_t < 2$  (see figure 3) and the periodic dependence of  $b_0$  on  $g_t$  [12] and consequent periodic dependence of the amplitude on  $g_t$ .

We have further recalled that perturbative study of the vertex correction leads to the expectation of insignificant modification of  $g_t$ ; however, we found no experimental constraints excluding a large modifications of  $g_t$ . Thus the most sensitive experiment to explore the value of  $g_t$  is at this juncture the  $h \rightarrow \gamma\gamma$  decay.

Non-perturbative standard model top structure comes to mind considering the remarkable top-Higgs minimal coupling  $e_{th} = m_t\sqrt{2}/v \simeq 1$ . We note the correct prediction of the Higgs mass based on vacuum (in)stability

argument [24]. Both these features suggest that the properties of the top quark are pivotal for the full understanding of the standard model.

A strong modification of  $g_t$  can arise from the beyond the standard model physics. As the highest mass particle, the top and its couplings are expected to be most sensitive to BSM, a point which will continue to motivate theoretical and experimental investigation of its potentially anomalous couplings, especially to strong [19] and electromagnetic and electroweak [2] gauge bosons, and magnetic moment in particular.

*Acknowledgments:* L.L. and J.R. thank the TH-division of the CERN Physics Department for hospitality while much of this work was done. This work was supported by a grant from the US Department of Energy, DE-FG02-04ER41318.

## References

- [1] W. Bernreuther, R. Bonciani, T. Gehrmann, R. Heinesch, T. Leineweber, P. Mastrolia and E. Remiddi, Nucl. Phys. B **706**, 245 (2005).
- [2] W. Bernreuther, R. Bonciani, T. Gehrmann, R. Heinesch, T. Leineweber, P. Mastrolia and E. Remiddi, Phys. Rev. Lett. **95**, 261802 (2005).
- [3] W. Bernreuther, J. Phys. G **35**, 083001 (2008).
- [4] J. F. Kamenik, M. Papucci and A. Weiler, Phys. Rev. D **85**, 071501 (2012)
- [5] D. Atwood, A. Kagan and T. G. Rizzo, Phys. Rev. D **52**, 6264 (1995)
- [6] P. Haberl, O. Nachtmann and A. Wilch, Phys. Rev. D **53**, 4875 (1996)
- [7] A. J. Larkoski and M. E. Peskin, Phys. Rev. D **83**, 034012 (2011)
- [8] S. J. Brodsky and S. D. Drell, Phys. Rev. D **22**, 2236 (1980).
- [9] M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Sov. J. Nucl. Phys. **30**, 711 (1979) [Yad. Fiz. **30**, 1368 (1979)].
- [10] W. J. Marciano, C. Zhang and S. Willenbrock, Phys. Rev. D **85**, 013002 (2012)
- [11] R. Angeles-Martinez and M. Napsuciale, Phys. Rev. D **85**, 076004 (2012)
- [12] J. Rafelski and L. Labun, arXiv:1205.1835 [hep-ph].
- [13] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B **106**, 292 (1976).
- [14] M. Reuter, in presentation at Marcel Grossmann 13, 3 July, 2012, Stockholm, Sweden.
- [15] G. Aad *et al.* [ATLAS Collaboration],
- [16] CMS presentation of July 4, 2012, see: <http://cms.web.cern.ch/news/observation-new-particle-mass-125-GeV> (Reference to be updated as soon as scientific report available.)
- [17] P. P. Giardino, K. Kannike, M. Raidal and A. Strumia,
- [18] A. Denner, S. Heinemeyer, I. Puljak, D. Rebuszi and M. Spira, Eur. Phys. J. C **71**, 1753 (2011)
- [19] R. Martinez, M. A. Perez and N. Poveda, Eur. Phys. J. C **53**, 221 (2008)
- [20] J. L. Hewett and T. G. Rizzo, Phys. Rev. D **49**, 319 (1994);
- [21] R. Martinez and J. A. Rodriguez, Phys. Rev. D **55**, 3212 (1997).
- [22] Z. Hioki and K. Ohkuma, Eur. Phys. J. C **65**, 127 (2010); Z. Hioki and K. Ohkuma, Eur. Phys. J. C **71**, 1535 (2011).
- [23] D. Atwood and A. Soni, Phys. Rev. D **45**, 2405 (1992).
- [24] C. D. Froggatt and H. B. Nielsen, Phys. Lett. B **368**, 96 (1996)